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On the fate of tachyonic quivers

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Abstract: We study gauge theories on the world-volume of D3-branes probing singularities. Seiberg duality can be realized as a sequence of Picard-Lefschetz monodromies on 3-cycles in the mirror manifold. In previous work, the precise meaning of gauge theories obtained by monodromies that do not correspond to Seiberg duality was unclear. Recently, it was pointed out that these theories contain tachyons, suggesting that the collection of branes at the singularity is unstable. We address this problem using (p, q) web techniques. It is shown that theories with tachyons appear whenever the (p, q) web contains crossing legs. A recent study of these theories with tachyons using exceptional collections proposed the notion of "well split condition." We show the equivalence between the well split condition and the absence of crossing legs in the (p, q) web. The (p, q) web has a natural resolution of crossing legs which was first studied in the construction of five dimensional fixed points using branes. Exploiting this result, we develop a generic procedure which we propose determines the quiver that corresponds to the stable bound state of D-branes that live on the singularity after the monodromy. This set is generically larger than the original set, meaning that there are extra massless gauge fields and matter fields in the quiver. Alternatively, one can argue that since these gauge and matter fields are initially assumed to be absent, the theory exhibits tachyonic excitations. We illustrate our ideas in an explicit example for D3-branes on a complex cone over dP_1, computing both the quiver and the superpotential.

Keywords: Duality in Gauge Field Theories, D-branes, Brane Dynamics in Gauge Theories.

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1. Introduction

There are several ways to engineer supersymmetric gauge theories using the low energy limit of String Theory. One of them is by probing singularities with a stack of D-branes. When a non-compact Calabi-Yau is probed with D3-branes, the result is an $\mathcal{N} = 1$, 3+1 dimensional gauge theory. More precisely, for every given singularity there is an infinite number of field theories that are related in the infrared by generalized Seiberg dualities [1].

When field theories are constructed in this way, it is possible to give a geometric interpretation to duality transformations. In particular, Seiberg duality can be obtained from Picard-Lefschetz (PL) monodromies of 3-cycles in the mirror manifold [2]–[4].

An interesting family of singularities is the one of complex cones over del Pezzo surfaces. The gauge theories associated to these geometries have been shown to exhibit a very rich physics, including chaotic RG flows and duality walls [5]–[8].

The computation of quiver theories for D-branes on singularities was substantially simplified in [9] using the (p, q) web techniques of [10, 11]. The methods in [9] showed how one can compute the quiver matrix using data which can be read from the (p, q) web which describes the geometry under study. Working in this context, it was realized in [3] that Seiberg duality transformations form a subgroup of the larger set of Picard-Lefschetz monodromies on (p, q) 7-branes. In particular it was demonstrated that there are PL monodromies which are not Seiberg duality. Furthermore, it was shown explicitly in this language how a single action of Seiberg duality is composed of few monodromies [12]. For this reason, theories that result from non-Seiberg monodromies were called *fractional Seiberg duals*. Few questions remain open: How are these fractional Seiberg duals related to the usual Seiberg duals? Do they fall into the same universality class in the infrared as for the case of ordinary Seiberg duals (i.e. in what sense, if any, are they dual)? Do two theories which are related by fractional Seiberg duality have the same moduli space of vacua? The same spectrum of gauge invariant operators? etc.

A prescription for deriving the quiver that results from fractional Seiberg duality is at hand. It uses intersections of 3-cycles in the mirror manifold. However, it is not clear how to compute the superpotential for such a quiver. This is another puzzle. Parallel discussions, in terms of exceptional collections, appeared subsequently in a series of papers by Herzog [13, 14].[1] The main focus of the work that followed [12] was on the identification of theories that are not Seiberg duals and on looking for a criterion that enables us to consistently restrict our attention to Seiberg dual theories, if it exists at all. It was argued in various examples that quivers for fractional Seiberg duals sometimes lead to IR theories with gauge invariant chiral operators of negative R-charge. That is, if one follows the same techniques as for usual Seiberg duals, such negative R-charges appear. One is lead to conclude that even though fractional Seiberg duals look like ordinary quiver gauge theories in the UV, they are in fact inconsistent or possibly incomplete.

Recently, an interesting observation about del Pezzo quivers was made by Aspinwall and Melnikov [18]. The authors emphasized a distinguishing characteristic of some of the quivers for the del Pezzo manifolds that turn out to be the fractional duals mentioned above. Some of the bifundamental fields in these models are induced by Ext^3 groups and are thus tachyonic.[2] The authors proved that a quiver is free of bifundamental tachyons if and only if $Ext^p(L_i, L_j) = 0$ for $i \neq j$ with $p \geq 3$. They also showed that one can get rid of the undesired tachyons by performing appropriate mutations (which is equivalent to the statement that these models are obtained from tachyon-free quivers by performing the inverse mutations). This certainly adds to the understanding of the gauge theories obtained from D-branes on complex cones over del Pezzo surfaces but raises natural questions, closely related to the ones we posed above. How should we interpret (if such an interpretation actually exists) the tachyonic quivers? Do they decay by the condensation of the tachyons or should we look for a different, stable configuration of D-branes at the singularity?

One can ask in what sense does a tachyon appear in such quiver theories? Does one of the bifundamental fields have a negative mass squared? This is hard to do since all of the bifundamental fields in a quiver theory that results from an exceptional collection are chiral. Therefore mass terms cannot be written for such fields. What is the gauge theory indication of an instability?

[1] A different perspective in the study of Seiberg duality was pursued in [15, 16, 17], where it was realized as a tilting equivalence of the quiver derived category.

[2] A related discussion of these states can be found in [14].

In this paper we will use the (p, q) web techniques of [9] to provide a new perspective into the problem of these tachyonic quivers or fractional Seiberg duals. As we will see, the webs associated to fractional dual theories are characterized by having *crossing external legs*. This seems to be one of the simplest ways of identifying these models. We will exploit the information in the webs further, and use them to give a concrete proposal for the stable quiver which results after the monodromy is performed. We will see that in these models, given a set of asymptotic (p, q) 7-branes, the minimal resolution of the corresponding singularity is of greater genus (here genus refers to the number of compact faces in the (p, q) web) than before applying the monodromy. That is, the appropriate stable set of branes corresponds to a quiver with more gauge groups and more matter fields than in the original one. This argument is general enough such that it can be applied to quiver theories for D3-branes on geometries with an arbitrary number of collapsing 4-cycles (as above, this number is simply given by the genus of the corresponding web) and not only to del Pezzos which have only one collapsing 4-cycle.

The organization of the paper is as follows. In section 2, we review the (p, q) web description of singularities. In section 3, we discuss the phenomenon of crossing legs in (p, q) webs and state our proposal for the stable quiver after monodromies. In section 4, we prove that the well split condition derived from exceptional collections to restrict the theories under study is actually equivalent to the absence of crossing legs in the associated web. section 5 shows how some seemingly unrelated difficulties in interpreting some quivers are in fact different manifestations of the same problem. Finally, guided by the perspective of crossing of external legs in (p, q) webs, we propose in section 6 that the stable set of D-branes for a fractional dual corresponds to a higher genus (p, q) web. We construct this higher genus quiver explicitly for a model obtained after performing a Picard-Lefschetz monodromy on a dP_1 quiver. We derive this theory using (p, q) webs machinery. There are no other straightforward methods to compute quivers for non-orbifold singularities with more than one collapsing 4-cycle. This gauge theory for a genus 2 (p, q) web is the main result of our paper. Finally, we discuss in section 7 the existence of negative R-charges in some fractional duals.

2. (p, q) web description of singularities

In [10, 11], (p, q) webs were introduced for studying five dimensional gauge theories with 8 supercharges. They are configurations of 5-branes in Type IIB string theory. The branes share 4+1 dimensions in which the field theory lives, and the physics is determined by the non-trivial configuration in a transverse plane.

There is an alternative interpretation of (p, q) webs as toric skeletons describing toric varieties. The precise correspondence was worked out in [20]. In this context, there is a T^2 fibration over every point of a web, with an S^1 going to zero size at each segment and the entire T^2 degenerating at the nodes. From this point of view, (p, q) webs are reciprocal to ordinary toric diagrams. We will use this correspondence in section 6.

Blowing-up a point (i.e. replacing it by a 2-sphere) corresponds in this language to replacing it by a segment, whose length is given by the size of the sphere. Different (p,q) webs can describe the same geometry. For example, for toric del Pezzo surfaces dP_n ($n \leq 3$), the SL$(3,\mathbb{C})$ symmetry of $\mathbb{P}^2$ can be used to place the blown-up points at any desired position.

The use of webs to describe geometries can be extended by attaching external legs to (p,q) 7-branes [21]. In this case, the legs no longer extend to infinity, permitting the study of cases that would otherwise have crossing legs. In addition, 7-branes make the global symmetries of the corresponding five dimensional field theories manifest. We present a typical (p,q) web example in figure 1.

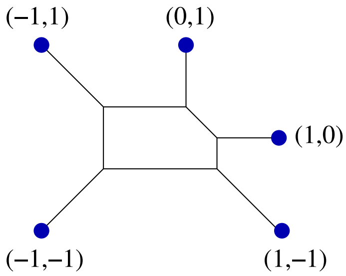

Figure 1: A typical (p,q) web. From a five dimensional point of view, it describes a supersymmetric gauge theory with 8 supercharges, an SU(2) gauge group, and 1 flavor. If interpreted as a toric skeleton, it corresponds to dP_2. The (p,q) charges of the external legs are shown. Blue circles indicate the (p,q) 7-branes at the ends of external legs.

In what follows, we will be interested in gauge theories on the world-volume of D3-branes probing singularities. The singularities we will focus on are complex cones over 2-complex dimensional toric compact manifolds Y.[3] These compact manifolds admit a (p,q) web description. The (p,q) webs encode the information about the degenerate fibers of the mirror manifold. This fact was exploited in [9], where a procedure for extracting the quiver of gauge theories in the world-volume of the D3-brane probes was developed. The number of gauge groups of the quiver theory is given by the Euler characteristic of Y, which in terms of the Betti numbers is given by

$$\chi(Y) = b_0 + b_2 + b_4 \tag{2.1}$$

$\chi(Y)$ can be immediately read from the (p,q) web and is given by the number of nodes. In addition, we will refer to the number of compact faces of a (p,q) web as its genus, which is equal to the number of collapsing 4-cycles.

For genus 1 webs, there is a one to one correspondence between web nodes (gauge groups) and external legs. The number of bifundamental fields between a pair of nodes i and j is given by the intersection of the corresponding 3-cycles in the mirror manifold, which in this case becomes

$$C_i \cdot C_j = \det \begin{pmatrix} p_i & q_i \\ p_j & q_j \end{pmatrix}. \tag{2.2}$$

We refer the reader to [9, 22, 23], for a more detailed explanation of the connection between (p,q) webs and toric geometry, and the computation of quivers on D-brane probes.

Picard-Lefschetz monodromy on the 3-cycles of the mirror geometry has a straightforward implementation as an action on the (p,q) charges of the 7-branes. The charges of a 7-brane that is moved around another one with charges (p_i,q_i) are modified by multiplying them by the monodromy matrix

$$M(p_i,q_i) = \begin{pmatrix} 1+p_iq_i & -p_i^2 \\ q_i^2 & 1-p_iq_i \end{pmatrix} \tag{2.3}$$

[3]In fact, although (p,q) webs have a natural interpretation in terms of toric geometry, their application extends to the determination of quivers for non-toric singularities, as shown in [9].

Operating on (p, q) 7-branes with (2.3) and its inverse is equivalent to acting with left and right mutations on the corresponding exceptional collections.

3. Brane crossing

The application of (p, q) web techniques to the determination of quiver gauge theories on D3-branes on singularities [9] identifies Seiberg duality as a Picard-Lefschetz monodromy [3]. The action of some Picard-Lefshetz monodromies leads to the phenomenon of *brane crossing*. Brane crossing was first observed in [10] when trying to construct a brane configuration for a five dimensional gauge theory with 8 supercharges that has more flavors than some critical value. figure 2 shows an attempt to construct a brane configuration for an SU(2) gauge theory with 5 flavors. An immediate consequence is the appearance of crossing branes. More generally, for a brane construction of an SU(n) gauge theory, if the number of flavors is more than $2n$ then the phenomenon of crossing branes is unavoidable.

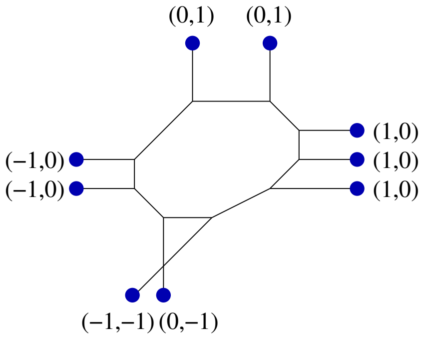

Figure 2: Brane crossing in an attempt to construct a (p, q) web for a five dimensional gauge theory with 8 supercharges, a gauge group SU(2), and 5 flavors in the fundamental representation. The genus of the curve is the number of compact faces in the web. Here it corresponds (before considering the addition of a face due to the crossing legs) to the rank of the SU(2) gauge group, namely 1.

Web diagrams indicate a possible solution to brane crossing. In fact, one can resolve the intersection point by replacing it by a segment, respecting the rules for the construction of a (p, q) web. After doing so, we obtain a (p, q) web whose genus has been increased. For the example of figure 2, a resolution of the (p, q) web diagram leads to the brane configuration of SU(3) gauge theory with 5 flavors as can be seen in figure 3.

From a five dimensional point of view, we start with a brane configuration describing an SU(2) gauge theory, namely with rank 1, and at some high vev for the scalar field we discover another brane which corresponds to an increase of the rank to 2, namely an SU(3) gauge group. Viewing this from the SU(3) gauge theory point of view, there are three adjoint vevs, ϕ_1, ϕ_2, ϕ_3 which satisfy the condition $\sum_i \phi_i = 0$. The two relevant scales are the differences $\phi_1 - \phi_2$ and $\phi_2 - \phi_3$. When one of these scales is much higher than the other scale, the effective theory can be described by an SU(2) gauge theory with a vev given by the smaller scale. This is the theory that we think we have at energies smaller than the high scale. However, once we start probing at energies comparable to the high scale, the

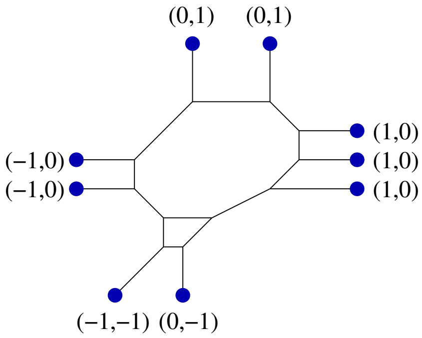

Figure 3: Web diagram originated from the configuration in figure 2 after resolving the intersection point. The genus of the new (p, q) web is 2.

SU(2) description is not valid anymore and we need to refer to the more complete SU(3) theory with 5 flavors. The SU(2) theory presents some instability which arises due to the fact that the description is lacking the additional states coming from the SU(3) theory.

This is the essence of our proposal. In what follows we will apply this principle to resolve quivers with an inconsistency — fractional Seiberg duals, ill split quivers, or tachyonic quivers — into consistent quivers by resolving their corresponding web diagram along the lines described in the previous paragraphs.

Given a quiver that corresponds to a (p, q) web with no brane crossing, we can perform a PL transformation. We have two cases to consider: If the resulting (p, q) web has no crossing legs then the associated quiver will correspond to a Seiberg dual on the node over which the monodromy was performed.[4] For such a case, we start with a consistent quiver and end up again with a consistent quiver. If the resulting (p, q) web has brane crossing like that of figure 2, then we need to resolve it and arrive at a (p, q) web with higher genus. The resolution is dictated by the rules of construction of (p, q) webs. Therefore we get a unique proposal for the resulting toric diagram and hence the resulting singular geometry. The new (p, q) web corresponds to a different toric diagram, a different singular manifold, and will give rise to a different quiver gauge theory. For this quiver theory, higher genus means more gauge groups and more matter fields.

In section 6, we will interpret this proposal from the point of view of the quiver gauge theory on D3-brane probes.

4. The well split condition from (p, q) webs: tachyons from crossing legs

In [12], it was realized that certain condition on a set of 7-branes guarantees that Seiberg duality in any node can be obtained by a sequence of PL monodromies. Exactly this same condition was identified in [13] in the context of exceptional collections, and was dubbed *well split* condition. Quivers that are not well split are labeled ill split. A well split quiver is such that for any node i, all the in-going nodes into i are placed to the left of i and all the out-going nodes, to the right of i, in the gauge theory exceptional collection $\mathcal{E}^{\mathcal{Q}}$ (we

[4] We want to remind the reader that, in some cases, Seiberg duality corresponds to a sequence of more than one PL monodromy [12].

follow the notation of [13] for collections).[5] That is

$$\begin{aligned} &C_i \cdot C_j > 0 \Rightarrow j \text{ to the left of } i \\ &C_i \cdot C_j < 0 \Rightarrow j \text{ to the right of } i\,. \end{aligned} \tag{4.1}$$

This condition has been recently revisited in [14], where a simple geometric picture was developed for it in terms of quivers. It relies on the identification of a special polygon in the quivers. Finding such a polygon is not trivial, and thus its existence for every well split quiver remains a conjecture. Here we give another, even simpler, geometric interpretation based on (p, q) webs, showing that (4.1) is indeed equivalent to the absence of crossing legs.

Let us recall that the number of bifundamental fields between two nodes is equal to the intersection between the corresponding 3-cycles in the mirror manifold and is given by the determinant in (2.2). Let us denote the non-trivial plane of the (p, q) web as the $x-y$ plane. Then, we can think of the charges of external legs as defining vectors in the $x-y$ plane. Equation (2.2) can be interpreted as the cross product between two charge vectors $\vec{C}_i$ and $\vec{C}_j$, which points in the z direction. The sign of the intersection (equal to the sign of the angle between the two vectors, θ_{ij}) just indicates whether the vector product points in the positive or negative z direction. In this language, well split quivers correspond to (p, q) webs such that, given any leg i, the rest of the legs can be separated into two sets In and Out, using the language of [12], one to the right and one to the left of i as we go around the web clockwise. These two sets are such that $\sin\theta_{ij} > 0$ for every j in In and $\sin\theta_{ij} < 0$ for every j in Out. This condition on angles between legs is clearly equivalent to the absence of crossing.

Let us pause for a moment to connect these ideas to other notions that have been developed for these quivers in the literature. In [14], the concept of $Ext^{1,2}$ was introduced. It turns out to be equivalent to the *strong helix* condition and also to the absence of Ext^3 tachyon generating maps. Furthermore, it was also shown that $Ext^{1,2}$ implies well split. Putting these ideas together with the discussion above, we have

$$\text{crossing legs} \Leftrightarrow \text{ill split} \Rightarrow \text{tachyonic } Ext^3\text{'s}\,. \tag{4.2}$$

We want to point out that the main virtue of interpreting the ill split quivers as arising from webs with crossing legs comes from the fact that the web diagram suggests how the quiver has to be modified in order to correspond to a stable configuration given some asymptotic 7-branes.

5. A new perspective on old puzzles

We summarize in this section a list of problematic quivers that have appeared in the literature in the past. Some of these gauge theories were discovered when looking at

[5] Along this paper we follow the convention that if $C_i \cdot C_j < 0$, then there is an arrow in the quiver from node i to node j. This notation differs from the one in [13] and has motivated the relabeling of the in-going and out-going sets of nodes. Both conventions are equivalent and are simply related by charge conjugation of all the fields.

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the effect of general Picard-Lefschetz monodromies on well behaved quivers. Others were originally constructed by a blowing-up procedure, trying to find the quivers on the world-volume of D3-branes probing non-toric del Pezzos. These problems appeared originally to be unrelated. Our current understanding indicates that the underlying common feature is that these quivers are tachyonic. We show that the presence of tachyons can be identified in all cases using the crossing leg criterion

5.1 Picard-Lefschetz monodromy versus Seiberg duality

Toric duality was discovered in [24, 25] as an ambiguity in the determination of the gauge theory on the world-volume of D3-branes probing a toric singularity. Later, it was realized that toric duals correspond to non-trivial realizations of Seiberg duality [3, 27]. Shortly after, the question of whether the group of Seiberg duality transformations on the nodes of the quiver can be extended to the larger one of Picard-Lefschetz monodromies (equivalently, general mutations in the language of exceptional collections) was raised [3, 12], and whether all the gauge theories obtained this way flow to the same universality class in the IR.

The first example of such a theory was found for the Zeroth Hirzebruch surface F_0 in [12]. The starting point was the theory given by the following set of (p,q) 7-branes

Brane	A	B	C	D
N_i	1	1	1	1
$[p_i, q_i]$	$[1,1]$	$[1,1]$	$[-3,-1]$	$[1,-1]$

(5.1)

Performing a Picard-Lefschetz monodromy of B around C, we arrive at the new set of charges

Brane	A	C	B	D
N_i	1	1	1	1
$[p_i, q_i]$	$[1,1]$	$[3,1]$	$[-5,-1]$	$[1,-1]$

(5.2)

The resulting webs are presented in figure 4.

Figure 5 shows the associated quivers. Although the quiver for this new model could be determined from the charges in (5.2) using the results of [9], the gauge theory was not fully specified, since the corresponding superpotential could not be determined. It is easy to verify that, if we require the theory to be conformal at the IR, not all closed loops in the quiver can appear together in the superpotential. Some of these terms must be absent in order to satisfy the vanishing of the beta function equations for their corresponding

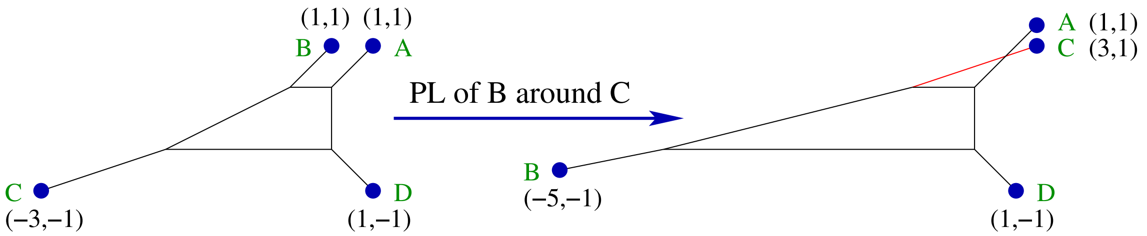

Figure 4: (p,q) webs for a normal and a fractional dual theories for F_0.

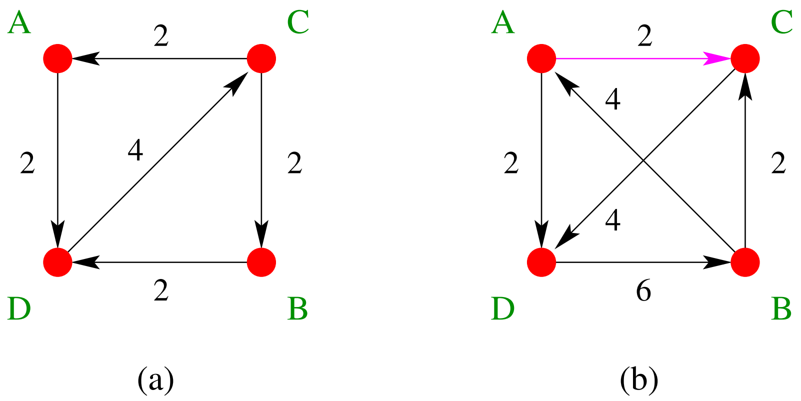

Figure 5: Quiver diagrams for a normal and a fractional dual theories for F_0. The tachyonic fields arising from the crossing legs in the web are indicated in magenta.

gauge and Yukawa couplings. The permitted loops can be identified by the requirement of conformal invariance. This by itself is not unusual and has been observed in other quiver theories. The new feature which is puzzling is that, in this case, it implies that some of the bifundamental fields cannot appear at all in any superpotential term. Furthermore, it was shown in [13] using exceptional collections that the R-charge of gauge invariants is given by $2(n+1)$, where n is the number of permutations of nodes with respect to the order they appear in the exceptional collection.

Using this information, even without knowing the precise form of the superpotential, one can conclude that, in some cases, conformal invariance requires some of the R-charges to be negative (this is not the case for the F_0 example in this section). Let us be more rigorous. The computation of R-charges using exceptional collections works only in those cases in which the flavor symmetry for multiple arrows is maximal, i.e. those examples in which the global symmetry is such that all fields connecting a given pair of nodes have the same R-charge. Superpotential interactions can break these flavor symmetries. Along the rest of the paper, we will work under the assumption that unknown superpotentials are such that fields in multiple arrows have equal R-charges. The fact that negative R-charges seem to be, in some cases, indicative of tachyonic quivers is very suggestive, but we have to keep in mind that this negative values might disappear once the actual global symmetries of the theory are taken into account. We will explore the possible appearance of negative R-charges in these models in section 7 using the exceptional collections approach. It is interesting to note that, even in the presence of negative R-charges, the value of a (one of the central charges of the superconformal algebra of the superconformal field theory) remains invariant. It is conjectured that a measures the number of degrees of freedom of the theory and obeys an analogue of the c-theorem in four dimensions.

5.2 Del Pezzo 7 and 8

Local mirror symmetry was used in [9] to obtain quivers for all del Pezzo surfaces dP_n. All the quivers were constructed using (p, q) webs, starting from dP_0 and blowing up an increasing number of points. Known quivers were recovered for the toric del Pezzos $n = 0$

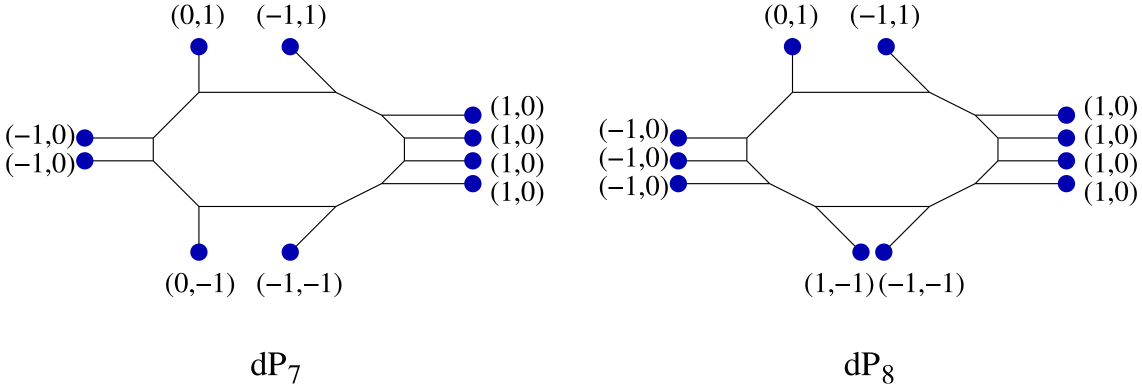

Figure 6: (p, q) webs constructed [9] for dP_7 and dP_8. They have crossing legs and are therefore ill split quivers.

to 3. In addition, new quivers were proposed for the first time for $n = 4$ to 8. The (p, q) webs constructed for $n = 6$, 7 and 8 had crossing external legs. In [28], it was verified that the quivers derived in [9] for dP_4 and dP_5 were correct, after obtaining their superpotentials using exceptional collections. Furthermore, [28] used an alternative 3-block quiver for dP_6 (whose (p, q) web description does not have crossing legs) and found its superpotential. A straightforward argument that shows that the interpretation of the dP_7 and dP_8 quivers of [9] is problematic is based on the values of R-charges. Computation of dibaryon R-charges determines that the minimum possible R-charge for bifundamental fields is equal to 1 for dP_7 and 2 for dP_8. Thus, the only possible superpotential for dP_7 would consists of quadratic mass terms, while it would be impossible to construct one for dP_8 [29]. Another fact is that none of these quivers can be connected by a sequence of Seiberg dualities with a healthy 3-block quiver, although they can be transformed into one by a sequence of PL monodromies.

We can now associate these difficulties, as was already noted in [31], to the fact that the corresponding webs have crossing external legs as shown in figure 6.

5.3 Four-block models for del Pezzos

Our last examples correspond to a nice set of 4-block quivers, from which the dP_1, dP_4 and dP_8 examples were studied in [18]. They can be constructed very easily using our (p, q) 7-brane techniques. The 7-brane backgrounds that correspond to the mirror of local dP_n surfaces are given by [9]

	A_1	$\ldots$	A_n	B	C	D
$[p_i, q_i]$	$[1, 0]$	$\ldots$	$[1, 0]$	$[2, -1]$	$[-1, 2]$	$[-1, -1]$

(5.3)

This configuration does not satisfy $\sum_i (p_i, q_i) = 0$ and thus does not correspond to an anomaly free quiver with all ranks equal to N. From this configuration, we can immediately construct another one that corresponds to a 4-block quiver with all gauge groups equal, by using PL monodromies to move the B brane all the way to the left of the n A_i branes.

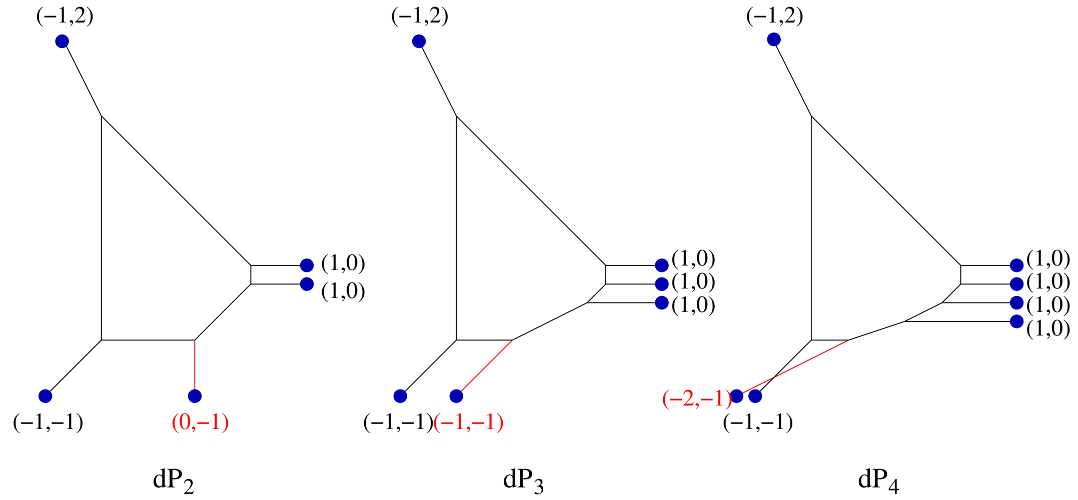

Figure 7: (p, q) webs for some 4-block models for dP_2 to dP_4. The dP_3 model in this family is actually a 3-block theory. Crossing legs appear for dP_n, with $n > 3$.

The (p, q) charge of A_i is $(1, 0)$ and its monodromy can be computed from (2.3) to be

$$M(1,0) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad M(1,0)^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}. \tag{5.4}$$

Then, the final collection of 7-branes becomes

	B	A_1	$\ldots$	A_n	C	D
N_i	1	1	$\ldots$	1	1	1
$[p_i, q_i]$	$[2-n, -1]$	$[1, 0]$	$\ldots$	$[1, 0]$	$[-1, 2]$	$[-1, -1]$

(5.5)

In order to illustrate how the crossing legs appear in these examples, we present in figure 7 the (p, q) webs for dP_2 to dP_4.

Labeling rows and columns in the order (B, A_i, C, D), we get the following intersection matrix

$$\mathcal{I} = \begin{pmatrix} 0 & 1 & 3-2n & n-3 \\ -1 & 0 & 2 & -1 \\ 2n-3 & -2 & 0 & 3 \\ 3-n & 1 & -3 & 0 \end{pmatrix} \tag{5.6}$$

which can be encoded in the quiver diagrams in figure 8.

From the (p, q) charges in (5.5), we see that the the crossing legs appear for $n > 3$. Furthermore, the intersection number between the crossing legs is

$$B \cdot D = n - 3 \tag{5.7}$$

in precise agreement with results from the computation of the dimensions of Ext^3's of [18].[6]

[6]The subsection in which this result appeared was removed in the second version of [18], but the computation remains valid.

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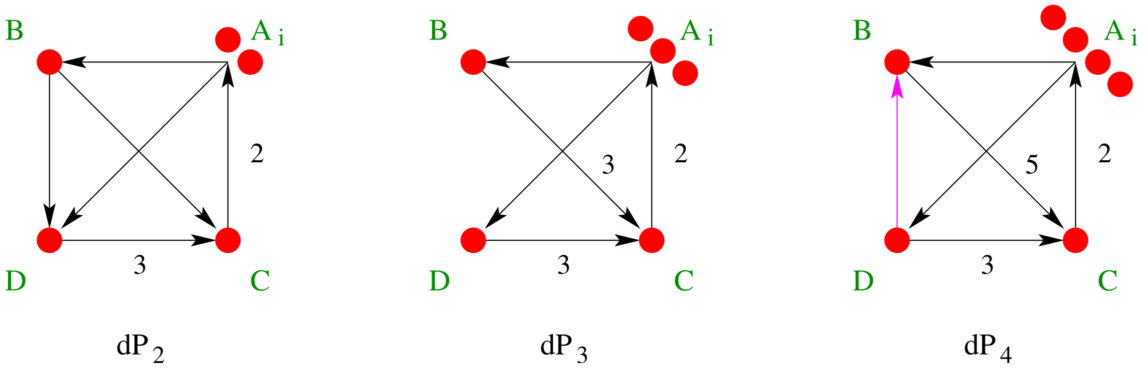

Figure 8: Quiver diagrams for the 4-block models whose webs are shown in figure 7. The tachyonic fields arising from the crossing legs in the web are indicated in magenta.

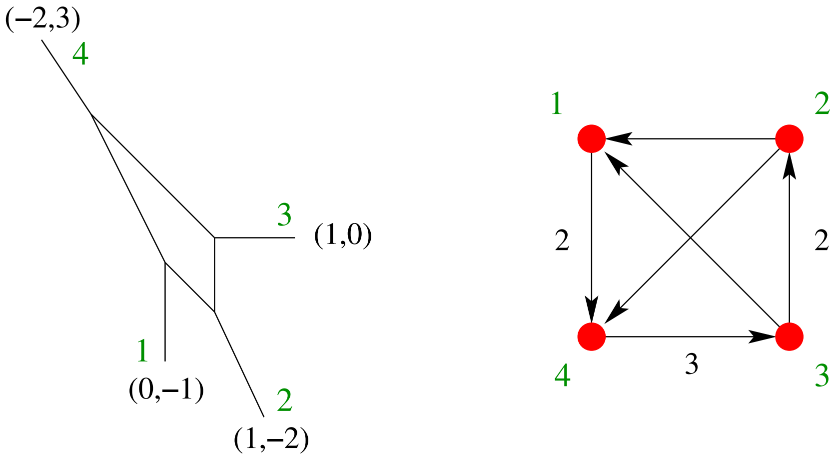

Figure 9: (p, q) web and quiver diagram for one of the phases of dP_1.

6. Getting rid of the tachyons: going to higher genus

We have discussed in section 4 how tachyonic fields can be identified by looking for crossing legs in the associated (p, q) webs. We illustrated this statement in section 5, in a large set of examples. According to the proposal we outlined in section 3, whenever we arrive at a configuration with crossing external legs by a PL monodromy transformation, the new set of stable branes corresponds to a higher genus resolution of the geometry. We now implement our proposal in an explicit example, computing the final quiver after the monodromy.

Let us start from the following set of (p, q) branes

Brane	1	2	3	4
N_i	1	1	1	1
$[p_i, q_i]$	$[0, -1]$	$[1, -2]$	$[1, 0]$	$[-2, 3]$

(6.1)

This configuration corresponds to dP_1. We show the (p, q) web and the associated quiver in figure 9.

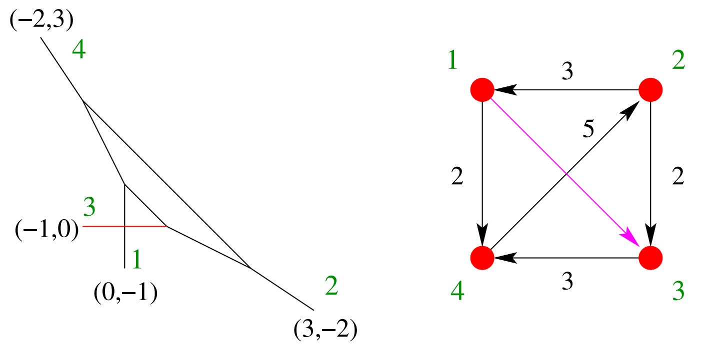

Figure 10: (p, q) web and quiver diagram obtained by PL monodromy of node 2 around 3 on the configuration in figure 9. The crossing of legs 1 and 3 give rise to tachyonic bifundamental fields between the corresponding nodes that we indicate in magenta.

Let us now perform a PL monodromy of node 2 to the right around 3. This transformation does not correspond to a Seiberg duality. The resulting configuration can be computed using (2.3) and is given by

Brane	1	3	2	4
N_i	1	1	1	1
$[p_i, q_i]$	$[0, -1]$	$[-1, 0]$	$[3, -2]$	$[-2, 3]$

(6.2)

These charges give rise to the following intersection matrix

$$\mathcal{I} = \begin{pmatrix} 0 & 3 & -1 & -2 \\ -3 & 0 & -2 & 5 \\ 1 & 2 & 0 & -3 \\ 2 & -5 & 3 & 0 \end{pmatrix} . \tag{6.3}$$

The resulting web and quiver are shown in figure 10. We have indicated in magenta the tachyonic bifundamental fields from the intersection of the crossing legs.

This monodromy transformation can be also seen as a mutation in the corresponding exceptional collection $\mathcal{E}^{\mathcal{Q}} = (E_1, E_2, E_3, E_4)$. The Chern characters of the initial sheaves $ch(E_i) = (r(E_i) + c_1(E_i) + ch_2(E_i))$ are

$$\begin{aligned} ch(E_1) &: (-1, H - E_1, 0) \qquad (6.4)\\ ch(E_2) &: \left(0, E_1, -\frac{1}{2}\right) \\ ch(E_3) &: \left(2, -H, -\frac{1}{2}\right) \\ ch(E_4) &: (-1, 0, 0) \qquad (6.5) \end{aligned}$$

Performing a left right mutation of E_2 around E_3, $E_2 \to R_{E_3} E_2$, we get

$$\begin{aligned} ch(E_1) &: (-1, H - E_1, 0) \\ ch(E_3) &: \left(-2, H, \frac{1}{2}\right) \end{aligned}$$

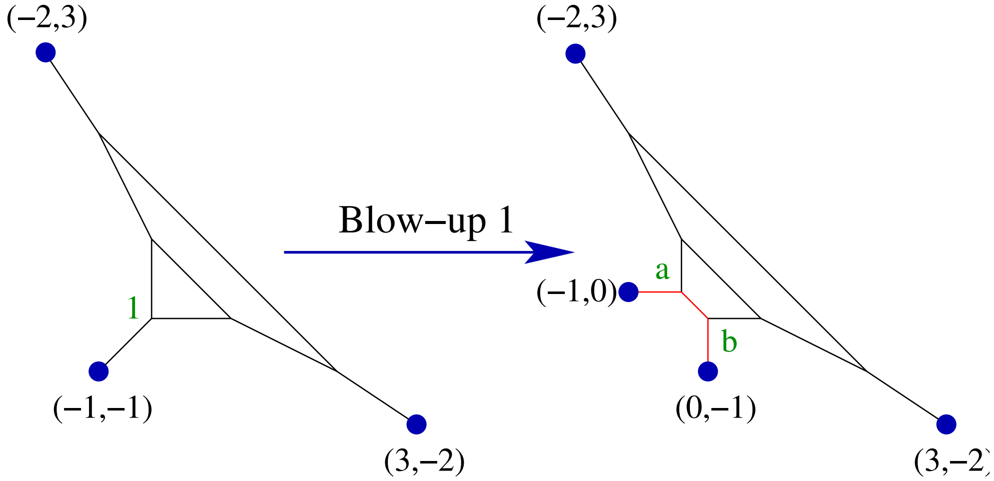

Figure 11: Connection by a blow-up between the (p,q) web for the $\mathbb{C}^3/\mathbb{Z}_5$ orbifold and the one for a fractional Seiberg dual of dP_1 corresponding to the resolution of the crossing legs.

$$\begin{aligned} ch(E_2) &: \left(4, -2H + E_1, -\frac{3}{2}\right) \\ ch(E_4) &: (-1, 0, 0)\,, \end{aligned} \tag{6.6}$$

where we have inverted the sign of $ch(E_2)$. The intersection matrix computed with these charges is in agreement with (6.3), computed using the (p,q) charges of equation (6.3).

The tachyonic quiver of figure 10 has to be replaced with a new, stable one. The stable quiver corresponds to a genus 2 (p,q) web that results after the external legs cross. The number of gauge groups is given by the number of nodes in the web. Therefore, we have six gauge groups in this case. The (p,q) web indicates that this theory corresponds to a blow-up of $\mathbb{C}^3/\mathbb{Z}_5$ as shown in figure 11.[7]

The gauge theory on D-branes probing an orbifold can be determined using the standard techniques of [32]. For D3-branes over the $\mathbb{C}^3/\mathbb{Z}_5$ orbifold, we have the quiver presented in figure 12 with superpotential

$$W = \sum_{i=1}^{5} (X_{i,i+2} Y_{i+2,i+4} Z_{i+4,i} - Y_{i,i+2} X_{i+2,i+4} Z_{i+4,i})\,, \tag{6.7}$$

where the nodes are identified modulo 5.

A geometric blow-up corresponds in the gauge theory on the brane probes to an unhiggsing [33]. Using figure 11, we can identify the correspondence between the nodes of the $\mathbb{C}^3/\mathbb{Z}_5$ and those of the new geometry. Node 1 of the orbifold comes from the higgsing of the nodes a and b. Furthermore, the blow-up condition implies that there is exactly one bifundamental field between these two nodes. Since the underlying geometry is an affine toric variety, every bifundamental field must appear exactly twice in the superpotential [24]. These observations determine that there are only two possible unhiggsed theories. This is

[7]The (p,q) web diagram for $\mathbb{C}^3/\mathbb{Z}_5$ can be found, for example, in [12].

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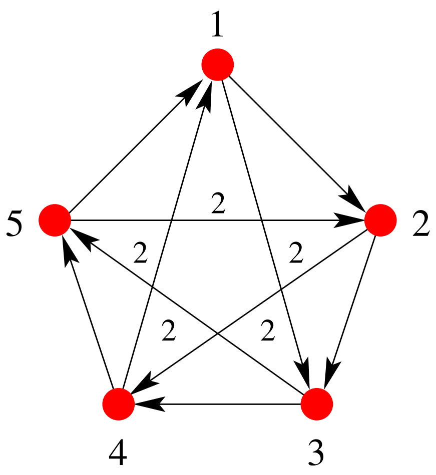

Figure 12: Quiver diagram for the $\mathbb{C}^3/\mathbb{Z}_5$ orbifold.

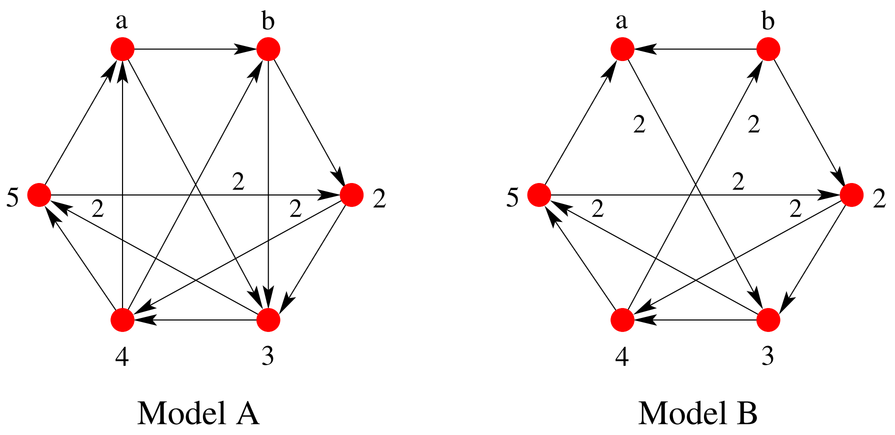

Figure 13: Quiver diagrams for the two inequivalent toric blow-ups of $\mathbb{C}^3/\mathbb{Z}_5$.

in accordance with the fact that there are only two possible inequivalent toric blow-ups of the $\mathbb{C}^3/\mathbb{Z}_5$ geometry. These blow-ups are shown in figure 14. The two quivers obtained in this way are presented in figure 13.

The superpotential for Model A is

$$\begin{aligned} W_A = {} & X_{52}Z_{23}Y_{35} - Y_{52}Z_{23}X_{35} + X_{24}Z_{45}Y_{52} - Y_{24}Z_{45}X_{52} + X_{35}X_{5a}X_{a3} - X_{a3}Z_{34}X_{4a} + \\ & + X_{b3}Z_{34}X_{4b} - X_{4b}X_{b2}X_{24} + X_{4a}X_{ab}X_{b2}Y_{24} - Y_{35}X_{5a}X_{ab}X_{b3} \end{aligned} \tag{6.8}$$

while the one for Model B is

$$\begin{aligned} W_B = {} & X_{35}Z_{5a}Y_{a3} - Y_{35}Z_{5a}X_{a3} + X_{24}Z_{45}Y_{52} - Y_{24}Z_{45}X_{52} + \\ & + X_{52}Z_{23}Y_{35} - Y_{52}Z_{23}X_{35} + X_{4b}Z_{b2}Y_{24} - Y_{4b}Z_{b2}X_{24} + \\ & + X_{a3}Z_{34}Y_{4b}X_{ba} - Y_{a3}Z_{34}X_{4b}X_{ba} \,. \end{aligned} \tag{6.9}$$

We observe that Model B has an SU(2) global symmetry, which is broken in Model A by the superpotential.

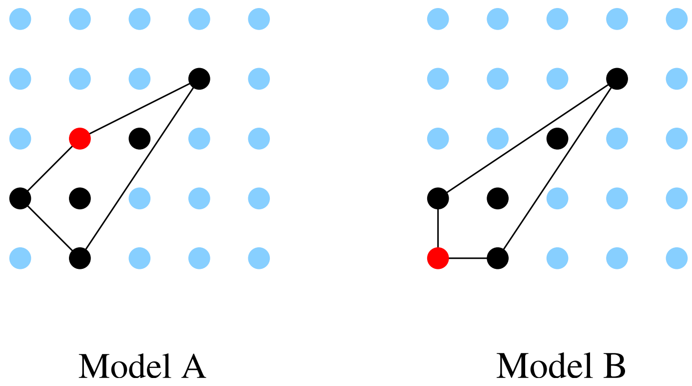

Figure 14: Toric diagrams for the two inequivalent blow-ups of $\mathbb{C}^3/\mathbb{Z}_5$. We have indicated the new node in the toric diagram in red.

It is straightforward to check that both quivers in figure 13, with their associated superpotentials given by (6.8) and (6.9), reduce to the ones of $\mathbb{C}^3/\mathbb{Z}_5$ by higgsing. In order to verify which of the gauge theories is the one that we are looking for, which corresponds to the second (p,q) web in figure 11, we compute their moduli spaces. The corresponding toric diagrams are shown in figure 14, from which we conclude that Model B is the theory we are pursuing.

7. Negative R-charges

We have discussed the existence of tachyons in fractional Seiberg dual quivers and its connection to the crossing legs in the associated (p,q) web. Let us explore now another characteristic signature of some of these theories.

A peculiarity of some of the fractional dual models is the existence of negative R-charges. This fact has already been noticed in the past [13]. Let us consider one of such examples. A practical way to compute R-charges of del Pezzo quivers, based solely on information in the quiver, was derived in [30] from exceptional collections. It is given by

$$R(X_{ij}) = \frac{2}{(9-n)d_i d_j} \times \begin{cases} S_{ij}^{-1} & \text{if } S_{ij}^{-1} \neq 0 \\ 2 - S_{ij}^{-1} & \text{if } S_{ij}^{-1} = 0\,, \end{cases} \tag{7.1}$$

where S is the incidence matrix of the quiver. S is upper triangular and such that $S_{ij} = \mathcal{I}_{ij}$ for $i < j$ and $S_{ii} = 1$. This method is closely related to the one based on the calculation of R-charges for dibaryon operators of [34]. An alternative, more general, procedure is to require the beta functions for gauge and superpotential couplings to vanish and then to fix the remaining freedom using the principle of a-maximization [35].

Let us apply (7.1) to the 4-block del Pezzo models discussed in section 5.3. The incidence matrix for these models is

$$S = \begin{pmatrix} 1 & -1 & 2n-3 & 3-n \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{7.2}$$

from which we obtain the following set of R-charges

$$
\begin{aligned}
&n \le 1 \; : \quad R_{BC} = \frac{5}{4} - \frac{1}{2n} > 0 \qquad && n > 1 \; : \quad R_{CB} = \frac{3}{4} + \frac{1}{2n} \\
&n \le 3 \; : \quad R_{BD} = \frac{5}{4n} - \frac{3}{4} > 0 \qquad && n > 3 \; : \quad R_{DB} = \frac{11}{4} - \frac{5}{4n} < 0 \\
&R_{CA} = \frac{1}{2} && R_{AD} = \frac{3}{4} \\
&R_{DC} = \frac{3}{4}
\end{aligned}
\tag{7.3}
$$

We see that R_{DB} becomes negative for $n > 3$, exactly when the D and B legs start crossing. In this case, gauge invariant dibaryon states with negative R-charge, and hence negative conformal dimension, can be constructed by antisymmetrizing N copies of the negative R-charge X^i_{DB} bifundamental fields. This fact would violate unitarity.

8. Conclusions

The precise significance of general Picard-Lefschetz monodromy transformations on 3-cycles that do not give rise to Seiberg dualities in the field theories on D3-branes probing the mirror manifold has been obscure in the past. The same problem has appeared as an unclear interpretation of general mutations in the exceptional collection approach to quivers for D-branes on singularities.

Based on the (p, q) web description of singularities we have developed a novel perspective into the tachyonic quivers that arise after fractional Seiberg duality. In section 4, we proved the equivalence between the ill split condition of exceptional collections and the crossing of external legs in the associated (p, q) web. This new interpretation makes the identification of tachyonic ill split quivers straightforward. We re-examined various previously problematic quivers in section 5 under this new light. Another advantage of this approach is that it indicates a sensible proposal for the set of stable marginally bound states of D-branes after monodromies are performed. This information is given by specifying the corresponding quiver and superpotential of a higher genus (p, q) web. For ill split quivers, the corresponding web gets an increase of the genus, i.e. an increase in the number of collapsing 4-cycles in the singularity.

In section 6, we applied our proposal to a specific example obtained after performing a Picard-Lefschetz monodromy on a dP_1 quiver. The new model corresponds to a genus 2 (p, q) web. Using the (p, q) web to relate the new theory to the $\mathbb{C}^3/\mathbb{Z}_5$ orbifold by a blow-up, we managed to determine both the quiver and its superpotential after the monodromy. As a bonus, the theories A and B considered in that section theory represent the first examples in which the gauge theories on a stack of D3-brane probes have been determined for non-orbifold toric singularities with more than one collapsing 4-cycles. We are hopeful that the techniques discussed in this paper will permit a systematic study of quivers for singularities of genus greater than one.

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A clear challenge is how to understand the genus increase and how to identify the stable quiver after monodromy within the frameworks of exceptional collection and of the derived category of quiver representations.

We hope to perform a detailed application of the ideas presented in this note in the near future.

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